Seat No. ________________

H-003-1164001

M. Sc. (Sem. IV) (CBCS) Examination

April - 2023

Mathematics : CMT-4001

(Linear Algebra)

Faculty Code : 003
Subject Code : 1164001

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions: (1) There are five questions.
(2) Answer all the questions.
(3) Each question carries 14 marks.

1 Answer any **seven** of the following: **7×2=14**

(1) Define (i) Singular linear transformation (ii) Regular linear transformation.

(2) Define characteristic root of a linear transformation.

(3) Define similar linear transformations and similar matrices for a finite dimensional vector space V over F.

(4) Let $T \in A_F(V)$. When the subspace W of V is invariant under T? Justify the answer with an example.

(5) Define (i) Nilpotent linear transformation (ii) Index of nilpotence of a nilpotent linear transformation.

(6) For A, B $\in F_n$, prove that tr (A + B) = tr (A) + tr (B).

(7) For A $\in \mathbb{C}_n$, prove that $(A^*)^* = A$.

(8) Define unitary liner transformation.

(9) Define bilinear form.

(10) Define homomorphism between two algebras A and B over a field F.

2 Answer any **two** of the following: **2×7=14**

(1) Let V be a finite dimensional vector space over a field F, then prove that $T \in A_F(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not zero.

(2) Let $\lambda \in F$ is a characteristic root of $T \in A_F(V)$, then prove that for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of q(T).

(3) Let V be a finite dimensional vector space over a field F and $T_1, T_2 \in A_F(V)$ are such that T_1 and T_2 are similar, then prove that there exists bases B_1, B_2 of V over F such that the matrix of T_1 in B_1 = the matrix of T_2 in B_2.

3 Answer the following **both** questions: **2×7=14**

(1) For A, B $\in F_n$, prove that (i) (A')' = A (ii) (A + B)' = A' + B' (iii) (AB)' = B'A'.

(2) Let V be an n-dimensional vector space over a field F and $T \in A_F(V)$. Let V_1 and V_2 are subspaces of V invariant under T. Let T_1 and T_2 are linear transformations induced by T on V_1 and V_2 respectively. If the minimal polynomial of T_1 over F is $p_1(x)$ while that of T_2 is $p_2(x)$, then prove that the minimal polynomial for T over F is the least common multiple of $p_1(x)$ and $p_2(x)$.

OR

3 Answer the following both questions: **2×7=14**

(1) Let A, B $\in F_n$ are similar then prove that det A = det B.

(2) Let $A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix} \in F_3$ then prove that the matrix is nilpotent and find the index of nilpotence.

4 Answer the following questions: **2×7=14**

(1) Find the solution using Cramer's rule

$$x + y + z = 6$$
$$y + 3z = 11$$
$$x - 2y + z = 0$$

(2) State and prove Jacobson's lemma.

5 Answer any **two** of the following : **2×7=14**

(1) Let V be a finite dimensional vector space over a field F. If $T \in A_F(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

(2) Let V be a finite dimensional vector space over a field F and $T \in A_F(V)$. Let $\lambda_1, \lambda_2, ..., \lambda_k$ are the distinct characteristic roots of T in F and $v_1, v_2 .... v_k$ are the characteristic vectors of T belonging to $\lambda_1, \lambda_2, ..., \lambda_k$ respectively, then prove that $v_1, v_2 .... v_k$ are linearly independent over F.

(3) Let (V, < >) be a finite dimensional inner product space over $\mathbb{C}$. Let $T \in A_{\mathbb{C}}(V)$ then prove that T is unitary if and only if $\langle T(u), T(u) \rangle = \langle u, u \rangle, \forall u \in V$.

(4) Prove that the determinant of a triangular matrix is equal to the product of its all the entries of the main diagonal.